KVN as a Pathfinder for the ngVLA

Richard Dodson¹
Maria Rioja¹,²,³

1- ICRAR/UWA
2- CASS/CSIRO
3- OAN/IGN
next generation VLA is the refresh of Radio Astronomy’s top observatory
next generation VLA is the refresh of Radio Astronomy’s top observatory.

Will connect the mm and sub-mm observations of ALMA to the cm regime.
next generation VLA is the refresh of Radio Astronomy’s top observatory

Will connect the mm and sub-mm observations of ALMA to the cm regime.

Will span 1-116GHz, therefore fill the role of SKA-High and SKA-mid!
ngVLA AKA SKA-High

next generation VLA is the refresh of Radio Astronomy’s top observatory

Will connect the mm and sub-mm observations of ALMA to the cm regime.

Will span 1-116GHz, therefore fill the role of SKA-High and SKA-mid!
next generation VLA is the refresh of Radio Astronomy’s top observatory

Will connect the mm and sub-mm observations of ALMA to the cm regime.

Will span 1-116GHz, therefore fill the role of SKA-High and SKA-mid!

The proposal to be submitted after the decadal review. But will not be under SKA-O.
ngVLA AKA SKA-High

Key Science Goals of ngVLA

Planetary Disks:
Follow on from ALMA, higher resolution
larger dust grains, lower frequencies & optical depth

Simulated ngVLA observations at 100 GHz – resolution 0.005°

Simulated ngVLA observations of protoplanetary disk continuum emission perturbed by a Jupiter mass planet at 5 AU (left), a 10 Earth mass planet at 5 AU (center), and a 30 Earth mass planet at 2.5 AU (right). The ngVLA observations at 100 GHz were simulated with 5 mas angular resolution and 0.5 μJy/beam rms. (Ricci et al. 2018)
ngVLA AKA SKA-High

Key Science Goals of ngVLA

Planetary Disks:
Follow on from ALMA, higher resolution
larger dust grains, lower frequencies & optical depth

Astro-Chemistry:
Focus on biogenic molecules, test chirality

A conservative simulation of 30 as-yet-undetected complex interstellar molecules (black) likely to be observed by the ngVLA above the confusion limit around hot cores with typical sizes of ~1" – 4". Key molecules are highlighted in color.
ngVLA AKA SKA-High

Key Science Goals of ngVLA

Planetary Disks:
Follow on from ALMA, higher resolution
larger dust grains, lower frequencies & optical depth

Astro-Chemistry:
Focus on biogenic molecules, test chirality

Galaxy Assembly:
Tracing gas content in CO, HI
Key Science Goals of ngVLA

Planetary Disks:
Follow on from ALMA, higher resolution, larger dust grains, lower frequencies, optical depth

Astro-Chemistry:
Focus on biogenic molecules, test chirality

Galaxy Assembly:
Tracing gas content in CO, HI

Pulsars:
In strong gravity regime frequency range allows views deep into G.C.
Key Science Goals of ngVLA

Planetary Disks:
Follow on from ALMA, higher resolution, larger dust grains, lower frequencies

Astro-Chemistry:
Focus on biogenic molecules, test chirality

Galaxy Assembly:
Tracing gas content in CO, HI

Pulsars:
In strong gravity regime frequency range allows views deep into G.C.

Black Holes:
Black Hole Hunter to detect the number of binary BHs. Compare with LIGO results
ngVLA AKA SKA-High

Key Science Goals of ngVLA

- **Planetary Disks:**
  - Follow on from ALMA, higher resolution
  - Larger dust grains, lower frequencies & optical depth

- **Astro-Chemistry:**
  - Focus on biogenic molecules, test chirality

- **Galaxy Assembly:**
  - Tracing gas content in CO, HI

- **Pulsars:**
  - In strong gravity regime frequency range allows views deep into G.C.

- **Black Holes:**
  - Black Hole Hunter to detect the number of binary BHs. Compare with LIGO results
Baseline ngVLA covers New Mexico (~500 to 1000km) with 214, 18m, offset-Gregorian antennas.

A dense core will cover the VLA site.

The `Long Baseline’ enhancement replaces the VLBA, providing continental baselines and sub-mas resolution.
Baseline ngVLA covers New Mexico (~500 to 1000km) with 214, 18m, offset-Gregorian antennas.

A dense core will cover the VLA site.

The `Long Baseline’ enhancement replaces the VLBA, providing continental baselines and sub-mas resolution.
Baseline ngVLA covers New Mexico (~500 to 1000km) with 214, 18m, offset-Gregorian antennas.

A dense core will cover the VLA site.

The `Long Baseline’ enhancement replaces the VLBA, providing continental baselines and sub-mas resolution.
ngVLA AKA SKA-High

Baseline ngVLA covers New Mexico (~500 to 1000km) with 214, 18m, offset-Gregorian antennas.

A dense core will cover the VLA site.

The `Long Baseline’ enhancement replaces the VLBA, providing continental baselines and sub-mas resolution.

On longer baseline the atmospheres (>VLA site) will be decorrelated - ngVLA is a VLBI machine.
Baseline ngVLA covers New Mexico (~500 to 1000km) with 214, 18m, offset-Gregorian antennas.

A dense core will cover the VLA site.

The `Long Baseline’ enhancement replaces the VLBA, providing continental baselines and sub-mas resolution.

On longer baseline the atmospheres (>VLA site) will be decorrelated - ngVLA is a VLBI machine.

KVN offers a good platform to investigate methods and technologies for the high frequencies.
Baseline ngVLA covers New Mexico (~500 to 1000km) with 214, 18m, offset-Gregorian antennas.

A dense core will cover the VLA site.

The `Long Baseline’ enhancement replaces the VLBA, providing continental baselines and sub-mas resolution.

On longer baseline the atmospheres (>VLA site) will be decorrelated - ngVLA is a VLBI machine.

KVN offers a good platform to investigate methods and technologies for the high frequencies.
Goal of the Comm. Study

High freq. long baseline interferometry is very interesting & very difficult

Sensitivity limitations come from high SEFD and short coherence times.
   Std. phase referencing (for increased sensitivity) impossible
Few sources can be studied (using self-calibration)

Astrometry is unachievable
   so relationship to other sources/freq., motion on the sky, etc. can not be derived

We have been tackling these (and other questions) to develop innovative calibration methods (see extra slide)
   Apply these to the ngVLA model
Correct the difficult mm-frequencies. Using phase solutions from easy lower cm-frequencies.

For non-dispersive (tropospheric) terms simply just scale. This skips a lot of details! Full solution is called Source/Frequency Phase Referencing (SFPR)

Two possible approaches:
- Fast Freq. Switching or Simultaneous Multi-band

Two Radio Interferometers:
- Very Long Baseline Array & Korean VLBI Network
Correct the difficult mm-frequencies. Using phase solutions from easy lower cm-frequencies.

For non-dispersive (tropospheric) terms simply just scale. This skips a lot of details! Full solution is called Source/Frequency Phase Referencing (SFPR)

Two possible approaches: Fast Freq. Switching or Simultaneous Multi-band

Two Radio Interferometers: Very Long Baseline Array & Korean VLBI Network
KVN Frequency Setup

Korean VLBI Network has an innovative optical system that allows simultaneous observations.

Beams from antenna

22, 43, 86, 129 GHz

Ellipsoidal Mirrors 1

43, 86, 129 GHz

Ellipsoidal Mirror 2

86, 129 GHz

Ellipsoidal Mirror 3

129 GHz

LPF1 22GHz

LPF2 43GHz

LPF3 86GHz 129GHz

22GHz

43GHz

86GHz

129GHz
KVN Frequency Setup

Korean VLBI Network has an innovative optical system that allows simultaneous observations in four bands.

Beams from the 22, 43, 86, and 129 GHz antennas.

Ellipsoidal Mirrors 1

Ellipsoidal Mirror 2

Ellipsoidal Mirror 3

KVN Yonsei

KVN Ulsan

KVN Tamna

22GHz

43GHz

86GHz

129GHz
VLBA Frequency Setup

In comparison:

Very Long Baseline Array has rapid switching between all the receivers.

Facilitates for lots of interesting science.

But also allows us to compare switching and simultaneous observational strategies.
Increased coherence times

The science case for simultaneous mm-wavelength receivers in radio astronomy

Richard Dodson\textsuperscript{a}, Maria J. Rioja\textsuperscript{a,b,c}, Taeyun Jung\textsuperscript{d,e}, Jose L. Gomez\textsuperscript{f}, Valentin Bujarrabal\textsuperscript{c}, Luca Moscadelli\textsuperscript{g}, James C.A. Miller-Jones\textsuperscript{h}, Alexandra J. Tetarenko\textsuperscript{i}, Gregory R. Sivakoff\textsuperscript{j}

\textsuperscript{a} International Centre for Radio Astronomy Research, The University of Western Australia, 35 Stirling Hwy, Australia
\textsuperscript{b} CSIRO Astronomy and Space Science, 26 Dick Perry Avenue, Kensington WA 6151, Australia
\textsuperscript{c} Observatorio Astronómico Nacional (IGN), Alfonso XII, 3 y 5, Madrid 28014, Spain
\textsuperscript{d} Korea Astronomy and Space Science Institute 776, Daejokdae-ro, Yuseong-gu, Daejeon 34055, Republic of Korea
\textsuperscript{e} University of Science and Technology, 217, Gaejeong-ro, Yuseong-gu, Daejeon 34113, Republic of Korea
\textsuperscript{f} Instituto de Astrofisica de Andalucía-CSIC, Glorieta de la Astronomía s/n, Granada E-18008, Spain
\textsuperscript{g} INAF-Osservatorio Astrofisico di Arcetri, Largo E. Fermi 5, 50125, Firenze, Italy
\textsuperscript{h} International Centre for Radio Astronomy Research, Curtin University, Perth, GPO Box U1987, WA 6845, Australia
\textsuperscript{i} Department of Physics, CCS 4-183, University of Alberta, Edmonton, AB T6G 2E1, Canada

\textbf{ARTICLE INFO}

Keywords:
Astronomical Instrumentation and techniques
Instrumentation: Interferometers
Methods: Observational
\textbf{ABSTRACT}

This review arose from the European Radio Astronomy Technical Forum (ERATec) meeting held in Firenze, October 2015, and aims to highlight the breadth and depth of the high-impact science that will be aided and assisted by the use of simultaneous mm-wavelength receivers.

Recent results and opportunities are presented and discussed from the fields of: continuum VLBI (observations of weak sources, astrometry, observations of AGN cores in spectral index and Faraday rotation), spectral line VLBI (observations of evolved stars and massive star-forming regions) and time domain observations of the flux variations arising in the compact jets of X-ray binaries.

Our survey brings together a large range of important science applications, which will greatly benefit from simultaneous observing at mm-wavelengths. Such facilities are essential to allow these applications to become more efficient, more sensitive and more scientifically robust. In some cases without simultaneous receivers the science goals are simply unachievable. Similar benefits would exist in many other high frequency astronomical fields of research.

(From Jung 12, Rioja 15, Dodson 17, Zhao 18)
Increased coherence times

Detection SNR for 3C279 at 129GHz for single baseline on KVN.

Freq. Phase Transfer provides good coherence times > 20min

Source/Freq. Phase Referencing, or FTP-Squared, does even better > 1 day

(From Jung 12, Rioja 15, Dodson 17, Zhao 18)
Increased coherence times

Detection SNR for 3C279 at 129GHz for single baseline on KVN.

Freq. Phase Transfer provides good coherence times > 20min

Source/Freq. Phase Referencing, or FTP-Squared, does even better > 1 day

(From Jung 12, Rioja 15, Dodson 17, Zhao 18)
Masers in AGB Stars (KVN)

Direct astrometric registration between (non-integer freq. ratio) maser transitions for R-Leo Min

22 (H$_2$O maser) to 42.8,43.1 (SiO masers) GHz around AGB star

Figure 3: The astrometric position of the AGB star RLMi (small cross), as derived from the centroid of the SiO J=0→1.

(From Dodson 2017, Yoon ’18 (Nature!))
Masers in AGB Stars (KVN)

Direct astrometric registration between (non-integer freq. ratio) maser transitions for R-Leo Min

22 (H$_2$O maser) to 42.8, 43.1 (SiO masers) GHz around AGB star VX Sgr

22 (H$_2$O maser) to 42.8, 43.1, 86, 129 GHz (SiO maser) around AGB star VX Sgr

(From Dodson 2017, Yoon ’18 (Nature!))
Masers in AGB Stars (KVN)

Direct astrometric registration between (non-integer freq. ratio) maser transitions for R-Leo Min

22 (H$_2$O maser) to 42.8, 43.1 (SiO masers) GHz around AGB star

22 (H$_2$O maser) to 42.8, 43.1, 86, 129 GHz (SiO maser) around AGB star VX Sgr

Dodson 2017, Yoon ’18 (Nature!)
Masers in AGB Stars (KVN)

Direct astrometric registration between (non-integer freq. ratio) maser transitions for R-Leo Min

22 (H₂O maser) to 42.8, 43.1 (SiO masers) GHz around AGB star VX Sgr

From Dodson 2017, Yoon '18 (Nature!)
Masers in AGB Stars (KVN)

Direct astrometric registration between (non-integer freq. ratio) maser transitions for R-Leo Min.

- 22 (H$_2$O maser) to 42.8, 43.1 (SiO masers) GHz around AGB star

Probing Astro-chemistry as ISM is seeded.
Phase Referencing between frequencies provides:
Registration of SiO (surrounding AGB) to water masers gives
3D structure of the archetypical Proto-Planetary Nebula

(From Dodson 18)
Phase Referencing between frequencies provides:
Registration of SiO (surrounding AGB) to water masers gives 3D structure of the archetypical Proto-Planetary Nebula

(From Dodson 18)
Phase Referencing between frequencies provides:
Registration of SiO (surrounding AGB) to water masers gives
3D structure of the archetypical Proto-Planetary Nebula

(From Dodson 18)
Standing Shocks in AGNs (VLBA)

Phase Referencing *purely* between frequencies:
To uncover the transition from B&K core-shift to unveil the standing shock, for BL-Lac

Figure 5: *a*) A sequence of simulated synchrotron total intensity images computed at, from the top, 86, 43, 22, 15, 12, 8, and 5-GHz, using a relativistic hydrodynamical model of a jet with a recollimation shock. *b*) Position of the simulated core as a function of frequency (black circles and line). The red curve indicates the best fit to the core positions between 5 and 22-GHz, which follows the conical Blandford & Königl jet model. However the 43 and 86-GHz data clearly deviate from the opacity core-shift curve, revealing the recollimation shock. *c*) Astrometric core-shifts of BL-Lac between 4.8 and 43-GHz, plotted as a function of frequency in black with 1-σ errors, adapted from Dodson et al. [32]. The Blandford & Koenigl model fitted to the cm-wavelength data (5 to 22-GHz, with $\kappa$ equal to -0.99 and $r_0$ equal to 5.3 mas GHz$^{-\kappa}$) is overlaid in red.

(From Dodson 2017, Molina 17)
Standing Shocks in AGNs (VLBA)

Phase Referencing *purely* between frequencies:
To uncover the transition from B&K core-shift to unveil the standing shock, for BL-Lac

**Figure 5**: a) A sequence of simulated synchrotron total intensity images computed at, from the top, 86, 43, 22, 15, 12, 8, and 5-GHz, using a relativistic hydrodynamical model of a jet with a recollimation shock. b) Position of the simulated core as a function of frequency (black circles and line). The red curve indicates the best fit to the core positions between 5 and 22-GHz, which follows the conical Blandford & Königl jet model. However the 43 and 86-GHz data clearly deviate from the opacity core-shift curve, revealing the recollimation shock. c) Astrometric core-shifts of BL-Lac between 4.8 and 43-GHz, plotted as a function of frequency in black with 1-σ errors, adapted from Dodson et al. [32]. The Blandford & Koenigl model fitted to the cm-wavelength data (5 to 22-GHz, with $\kappa$ equal to -0.99 and $r_0$ equal to 5.3 mas GHz$^{-\kappa}$) is overlaid in red.

(From Dodson 2017, Molina 17)
Standing Shocks in AGNs (VLBA)

Phase Referencing *purely* between frequencies:
To uncover the transition from B&K core-shift to unveil the standing shock, for BL-Lac

Figure 5: a) A sequence of simulated synchrotron total intensity images computed at, from the top, 86, 43, 22, 15, 12, 8, and 5-GHz, using a relativistic hydrodynamical model of a jet with a recollimation shock. b) Position of the simulated core as a function of frequency (black circles and line). The red curve indicates the best fit to the core positions between 5 and 22-GHz, which follows the conical Blandford & Königl jet model. However the 43 and 86-GHz data clearly deviate from the opacity core-shift curve, revealing the recollimation shock. c) Astrometric core-shifts of BL-Lac between 4.8 and 43-GHz, plotted as a function of frequency in black with 1-σ errors, adapted from Dodson et al. [32]. The Blandford & Koenigl model fitted to the cm-wavelength data (5 to 22-GHz, with $\kappa$ equal to -0.99 and $r_0$ equal to 5.3 mas GHz$^\kappa$) is overlaid in red.

(From Dodson 2017, Molina 17)
Standing Shocks in AGNs (VLBA)

Phase Referencing *purely* between frequencies: To uncover the transition from B&K core-shift to unveil the standing shock, for BL-Lac

![MHD Simulations](image1)

- Predict deviation for B&K optical depth core-shift model
- Perfect match to observations

Figure 5: (a) A sequence of simulated synchrotron total intensity images computed at, from the top, 86, 43, 22, 15, 12, 8, and 5-GHz, using a relativistic hydrodynamical model of a jet with a recollimation shock. (b) Position of the simulated core as a function of frequency (black circles and line). The red curve indicates the best fit to the core positions between 5 and 22-GHz, which follows the conical Blandford & Königl jet model. However the 43 and 86-GHz data clearly deviate from the opacity core-shift curve, revealing the recollimation shock. (c) Astrometric core-shifts of BL-Lac between 4.8 and 43-GHz, plotted as a function of frequency in black with 1-σ errors, adapted from Dodson et al. [32]. The Blandford & Königl model fitted to the cm-wavelength data (5 to 22-GHz, with $\kappa$ equal to -0.99 and $r_0$ equal to 5.3 mas GHz$^{-\kappa}$) is overlaid in red.

*(From Dodson 2017, Molina 17)*
Standing Shocks in AGNs (VLBA)

Phase Referencing *purely* between frequencies:
To uncover the transition from B&K core-shift to unveil the standing shock, for BL-Lac

MHD Simulations Predict deviation for B&K optical depth core-shift model

Made possible by VLBA freq agility:
Fast switching between 22/43/86GHz
Slower switching between 22/6/1.4GHz

(From Dodson 2017, Molina 17)
Standing Shocks in AGNs (VLBA)

Phase Referencing *purely* between frequencies:
To uncover the transition from B&K core-shift to unveil the standing shock, for BL-Lac

Made possible by VLBA freq agility:
Fast switching between 22/43/86GHz
Slower switching between 22/6/1.4GHz

(From Dodson 2017, Molina 17)
Jet Physics of X-ray binaries (VLA)

Some signals change very fast …
Shown are VLA observations of V404 Cygni in out-burst.

Sub-minute differences between the light-curves carries information on the jet-width as a function of distance down the jet.

Only sim observations will be able to follow this at mm-frequencies.

(From Tetarenko `17)
Some signals change very fast …
Shown are VLA observations of V404 Cygni in out-burst.

Sub-minute differences between the light-curves carries information on the jet-width as a function of distance down the jet.

Only sim observations will be able to follow this at mm-frequencies
ngVLA simulation results

What are the observational losses from fast freq. switching?

Characterising the performance of switching

Use delays and rates to predict the next solution

Scale for freq. and find no. > than 1/2 cycle error

Thermal noise-free case, but corrected phases are noisy
What are the observational losses from fast freq. switching?

Characterising the performance of switching

Use delays and rates to predict the next solution

Scale for freq. and find no. > than 1/2 cycle error

Thermal noise-free case, but corrected phases are noisy
What are the observational losses from fast freq. switching?

Characterising the

Thermal noise-free case, but corrected phases are noisy
What are the observational losses from fast freq. switching?

Characterising the performance of switching?

Use delays and rates to predict the next solution.

Scale for freq. and find no. > than 1/2 cycle error.

**Thermal noise-free case, but corrected phases are noisy.**

Over plotted are Flux Recovery.
ngVLA simulation results

What are the observational losses from fast freq. switching?

Characterising the performance of switching

Use delays and rates to predict the next solution

Scale for freq. and find no. > than 1/2 cycle error

20s

30s

40s

60s

Thermal noise-free case, but corrected phases are noisy

Over plotted are Flux Recovery + real results
What are the observational losses from fast freq. switching?

Characterising the performance of switching
Use delays and rates to predict the next solution
Scale for freq. and find no. > than 1/2 cycle error

Thermal noise-free case, but corrected phases are noisy
Over plotted are Flux Recovery + real results
SFPR works best when the frequency ratio (of the spectral reference point) is an integer.

\[ \Delta \theta_{\text{SFPR}} = \theta_{\text{high}} - \theta_{\text{low}} \times R \]

\[ \Delta \theta_{\text{SFPR}} = \theta_{\text{struct}} - 2\pi N \times R \]

So — to avoid problems best that

R is **INTEGER** or N is **ZERO**

We don’t want limited frequency coverage, so we should ensure we can track the fringe phase.

That is phase rate < 3E-13 for 100GHz & 30sec cycle

Typical AllanStdDev 1E-13 — will loose good fraction of data

Best solution is cycle time is zero .. but that is not the ngVLA design
SFPR limitation in VLBI

SFPR works best when the frequency ratio (of the spectral reference point) is an integer.

\[ \Delta \theta_{\text{SFPR}} = \theta_{\text{high}} - \theta_{\text{struct}} \]

So — to avoid problems, R is INTEGER or N is ZERO.

We don’t want limited frequency coverage, so we should ensure we can track the fringe phase.

That is phase rate < 3E-13 for 100GHz & 30sec cycle.

Typical AllanStdDev 1E-13 — will loose good fraction of data.

Best solution is cycle time is zero .. but that is not the ngVLA design.
SFPR limitation in VLBI

SFPR works best when the frequency ratio (of the spectral reference point) is an integer:

$$\Delta \theta_{\text{SFPR}} = \theta_{\text{high}}$$

$$\Delta \theta_{\text{SFPR}} = \theta_{\text{struct}}$$

So — to avoid problems, \( R \) is INTEGER or \( N \) is ZERO.

We don’t want limited frequency coverage, so we should ensure we can track the fringe phase.

That is phase rate < 3E-13 for 100GHz & 30sec cycle
Typical AllanStdDev 1E-13 — will lose good fraction of data
Best solution is cycle time is zero .. but that is not the ngVLA design.
SFPR limitation in VLBI

SFPR works best when the frequency ratio (of the spectral reference point) is an integer:

$$\Delta \theta_{\text{SFPR}} = \theta_{\text{high}}$$

$$\Delta \theta_{\text{SFPR}} = \theta_{\text{struct}}$$

So — to avoid problems, $R$ is an integer or $N$ is zero.

We don’t want limited frequency coverage, so we should ensure we can track the fringe phase.

That is phase rate $< 3E-13$ for 100GHz & 30sec cycle
Typical AllanStdDev 1E-13 — will lose good fraction of data
Best solution is cycle time is zero .. but that is not the ngVLA design
Possible StrawMan System (?)

Freq. Selective Grating @ 50GHz

<table>
<thead>
<tr>
<th>Band</th>
<th>( f_L ) (GHz)</th>
<th>( f_M ) (GHz)</th>
<th>( f_H ) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>6.6</td>
<td>12.3</td>
</tr>
<tr>
<td>3</td>
<td>12.3</td>
<td>15.9</td>
<td>20.5</td>
</tr>
<tr>
<td>4</td>
<td>20.5</td>
<td>26.4</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>30.5</td>
<td>39.2</td>
<td>50.5</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
<td>90.1</td>
<td>116</td>
</tr>
</tbody>
</table>

Quasi-optical system from KVN allows separation of freq. bands and therefore simultaneous co-observation.

Very fast switching (5sec) with precise timing may also work

Both allow precise calibration and therefore the target science.
Conclusions:

Simultaneous multifreq observations allows for more and better science

ngVLA performance will benefit greatly from this configuration

Loss of signal == Fraction of observing time

Taking this in to account the costs for Multi-Freq receiver are minimal in comparison

We continue to strongly recommend that this option is fully explored; our experience favours the KVN-style over the VLBA-style
# Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td></td>
<td><strong>Asaki ’07 + Rioja ’11</strong></td>
</tr>
<tr>
<td></td>
<td>( \frac{\nu}{8,\text{GHz}} \frac{\Delta \ell}{3,\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(45^\circ)} \right] )</td>
<td>( 5C_w \frac{\nu}{8,\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \left[ \frac{T_{swt}}{60,\text{s}} + 0.16 \frac{\Delta \theta}{2^\circ \sec(45^\circ)} \right]^{5/6} )</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>( 14.5 \left( \frac{\nu}{8,\text{GHz}} \right)^{-1} \frac{\Delta I}{6,\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right] )</td>
<td>( 2.5 \left( \frac{\nu}{8,\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{swt}}{60,\text{s}} \right]^{5/6} )</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at \(~10\text{km}\) and Ionosphere at \(~100\text{km}\) (<8GHz)
Both dynamic (fast changing) and static (slow changing) terms
Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time
Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere</td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta \ell}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(45^\circ)} \right]$</td>
<td>$5C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \left[ \frac{T_{swl}}{60s} + 0.16 \frac{\Delta \theta \sec(Z)}{2^\circ \sec(45^\circ)} \right]^{5/6}$</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta I}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{swl}}{60s} \right]^{5/6}$</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz)

Both dynamic (fast changing) and static (slow changing) terms

Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time
# Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Asaki ‘07 + Rioja ‘11</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta \ell}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(45^\circ)} \right]$</td>
<td>$5 C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \left[ \frac{T_{\text{swl}}}{60s} + 0.16 \frac{\Delta \theta}{2^\circ \sec(45^\circ)} \right]^{5/6}$</td>
<td></td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta I}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{\text{swl}}}{60s} \right]^{5/6}$</td>
<td></td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz)

Both dynamic (fast changing) and static (slow changing) terms

Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time
## Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td>$14.5 \frac{\nu}{8 \text{GHz}} \frac{\Delta \ell}{3 \text{cm}} \left[ \frac{\Delta \theta \cos(41^\circ)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$5C_\nu \frac{\nu}{8 \text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \frac{T_{\text{swl}}}{60s} + 0.16 \frac{\Delta \theta}{2^\circ} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \left[ \frac{T_{\text{swl}}}{60s} \right]^{5/6}$</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>$14.5 \frac{\nu}{8 \text{GHz}} \frac{\Delta I}{6 \text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \frac{\nu}{8 \text{GHz}} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{\text{swl}}}{60s} \right]^{5/6}$</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz)

Both dynamic (fast changing) and static (slow changing) terms

Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time
### Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta \ell}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ)}{2^\circ} \frac{\tan(Z)}{\tan(45^\circ)} \right]$</td>
<td>$5C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \left[ \frac{T_{swl}}{60\text{s}} + 0.16 \frac{\Delta \theta}{2^\circ} \frac{\sec(Z)}{\sec(45^\circ)} \right]^{5/6}$</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta I}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ)}{2^\circ} \frac{\tan(Z)}{\tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta}{2^\circ} \frac{\sec(Z)}{\sec(43^\circ)} + 0.21 \frac{T_{swl}}{60\text{s}} \right]^{5/6}$</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at \( \sim 10\text{km} \) and Ionosphere at \( \sim 100\text{km} \) (<8GHz)

Both dynamic (fast changing) and static (slow changing) terms

Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time
### Functional analysis of residual contributions

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz). Both dynamic (fast changing) and static (slow changing) terms are dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time.

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Troposphere</td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta \ell}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ)}{2^\circ} \frac{\tan(Z)}{\cos(Z) \tan(45^\circ)} \right]$</td>
<td>$5C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(15^\circ)}^{1/2} \left[ \frac{T_{swl}}{60s} + 0.16 \frac{\Delta \theta}{2^\circ} \frac{\sec(Z)}{\sec(15^\circ)} \right]^{5/6}$</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta I}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ)}{2^\circ} \frac{\tan(Z)}{\cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{swl}}{60s} \right]^{5/6}$</td>
</tr>
</tbody>
</table>
### Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta \ell}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(45^\circ)} \right]$</td>
<td>$5C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)}^{1/2} \left[ \frac{T_{swl}}{60\text{s}} + 0.16 \frac{\Delta \theta}{2^\circ \sec(15^\circ)} \right]^{5/6}$</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta \ell}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)}^{1/2} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{swl}}{60\text{s}} \right]^{5/6}$</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz)

Both dynamic (fast changing) and static (slow changing) terms

Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time
# Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Dynamic Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta l}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(45^\circ)} \right]$</td>
<td>$5C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)} \frac{1}{2^\circ} \left[ T_{swt} + 0.16 \frac{\Delta \theta \sec(Z)}{2^\circ \sec(45^\circ)} \right]^{5/6}$</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta I}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)} \frac{1}{2^\circ} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 \frac{T_{swt}}{60\text{s}} \right]^{5/6}$</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz)
Both dynamic (fast changing) and static (slow changing) terms
Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time

Asaki '07 + Rioja '11
Functional analysis of residual contributions

<table>
<thead>
<tr>
<th>Residual phase errors (deg)</th>
<th>Static Contributions</th>
<th>Asaki '07 + Rioja '11</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Troposphere</strong></td>
<td>$14 \frac{\nu}{8\text{GHz}} \frac{\Delta \ell}{3\text{cm}} \left[ \frac{\Delta \theta \cos(45^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(45^\circ)} \right]$</td>
<td>$5C_w \frac{\nu}{8\text{GHz}} \frac{\sec(Z)}{\sec(45^\circ)} \left[ \frac{T_{\text{swl}}}{60s} + 0.16 \frac{\Delta \theta \sec(Z)}{2^\circ \sec(45^\circ)} \right]^{5/6} + 2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>$14.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\Delta I}{6\text{TECU}} \left[ \frac{\Delta \theta \cos(41^\circ) \tan(Z)}{2^\circ \cos(Z) \tan(41^\circ)} \right]$</td>
<td>$2.5 \left( \frac{\nu}{8\text{GHz}} \right)^{-1} \frac{\sec(Z)}{\sec(43^\circ)} \left[ \frac{\Delta \theta \sec(Z)}{2^\circ \sec(43^\circ)} + 0.21 T_{\text{swl}} \right]^{5/6}$</td>
</tr>
</tbody>
</table>

Atmosphere has Troposphere at ~10km and Ionosphere at ~100km (<8GHz)

Both dynamic (fast changing) and static (slow changing) terms

Dependent on Angular Sep, Residual Zenith path, Residual TEC and Switching time