전파 간섭계

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서울대학교 평창캠퍼스
2019 전파 여름학교

권우진
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Radio observations

- To achieve 1 arc-second resolution
  at λ = 500 nm: D ~ 10 cm
  at λ = 1 mm : D ~ 200 m

$\theta \sim \lambda/D$

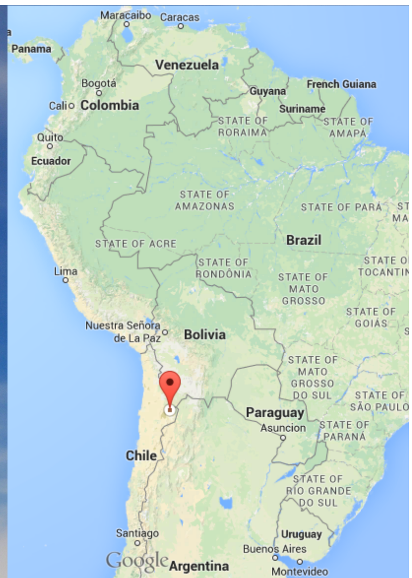
- Difficulties in building a big radio telescope:
  1. The required tracking accuracy ~ $\theta/10$ but the best mechanical tracking and pointing accuracy ~ 1” due to
     - Gravitational sagging
     - Antenna deformations caused by differential solar heating
     - Wind gusts
  2. Surface accuracy ~ $\lambda/20$

What radio interferometers look like?

- Arrays: e.g., JVLA, SMA, NOEMA, ALMA
- Very Long Baseline Interferometers: e.g., KVN

Atacama **L**arge **M**illimeter/submillimeter **A**rray

- The largest ground-based astronomical facility
- 50 12-m, 12 7-m, 4 12-m = 66 antennas
- ~5000 m in altitude, Chajnantor plateau, Chile
- East Asia, Europe, North America, & Chile
- https://almascience.org



ALMA 인류 역사상 가장 규모가 큰 천문대

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References

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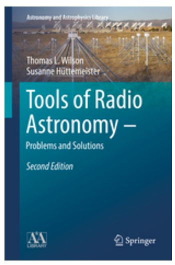
  https://science.nrao.edu/opportunities/courses/era/

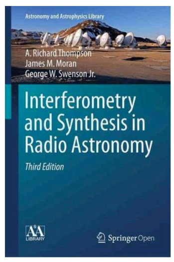
- Fundamentals of Radio Astronomy (ch. 5, 6)
  J. M. Marr, R. L. Snell, and S. E. Kurtz

- Tools of Radio Astronomy
  K. Rohlfs and T. L. Wilson

- Interferometry and Synthesis in Radio Astronomy
  A. Richard Thompson, James M. Moran, and George W. Swenson, Jr.

- Synthesis Imaging in Radio Astronomy
  Astronomical Society of the Pacific Conference Series (Volume 180)





Fourier transform

$$F(s) \equiv \int_{-\infty}^{\infty} f(x) \quad e^{-2\pi i s x} \, dx,$$

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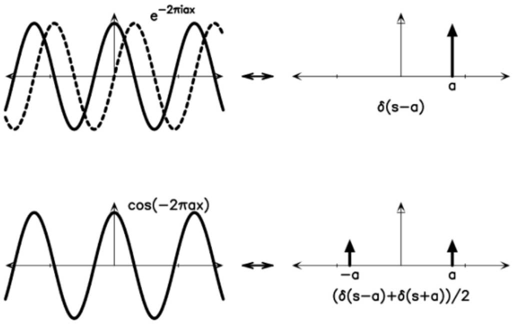

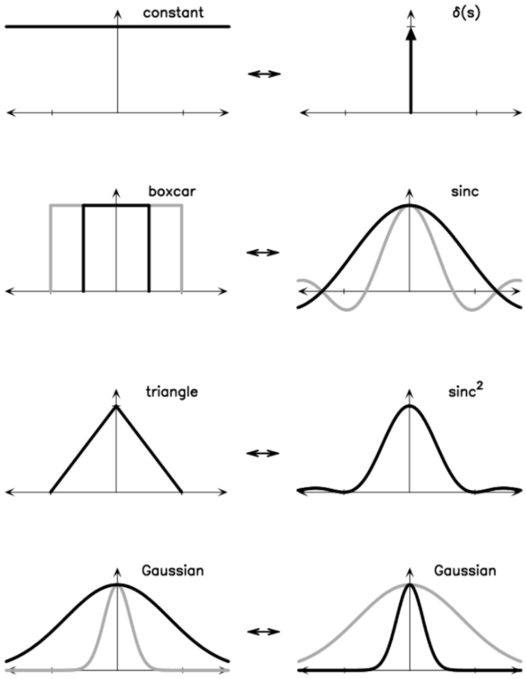

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$$f(x) + g(x) \Leftrightarrow F(s) + G(s).$$

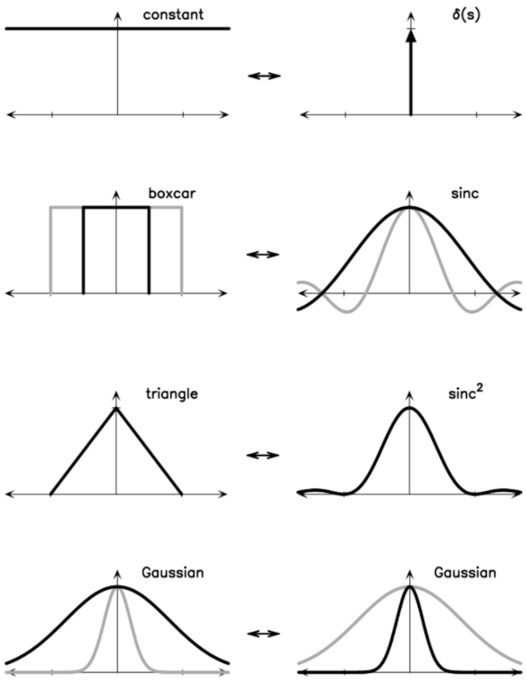
$$af(x) \Leftrightarrow aF(s).$$

$$f(x-a) \Leftrightarrow e^{-2\pi i a s} F(s).$$

$$f(ax) \Leftrightarrow \frac{F(s/a)}{|a|}.$$

$$f(x)\cos(2\pi\nu x) \Leftrightarrow \frac{1}{2}F(s-\nu) + \frac{1}{2}F(s+\nu).$$

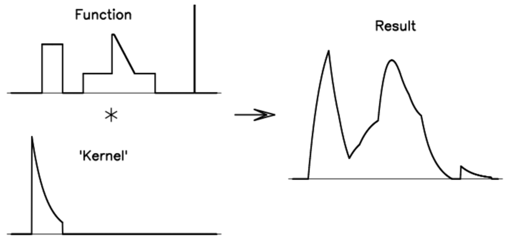
$$\frac{df}{dx} \Leftrightarrow i2\pi s F(s).$$



Convolution & Cross-correlation

- Convolution

$$h(x) = f * g \equiv \int_{-\infty}^{\infty} f(u)\, g(x-u) \quad du.$$



$\boxed{f * g \Leftrightarrow F \cdot G.}$ Convolution theorem

- Cross-correlation

$$f \star g \equiv \int_{-\infty}^{\infty} f(u)\, g(u-x) \quad du.$$

$\boxed{f \star g \Leftrightarrow \overline{F} \cdot G.}$ Cross-correlation theorem

Auto-correlation

$$f \star f \Leftrightarrow \overline{F} \cdot F = |F|^2.$$

Wiener-Khinchin theorem

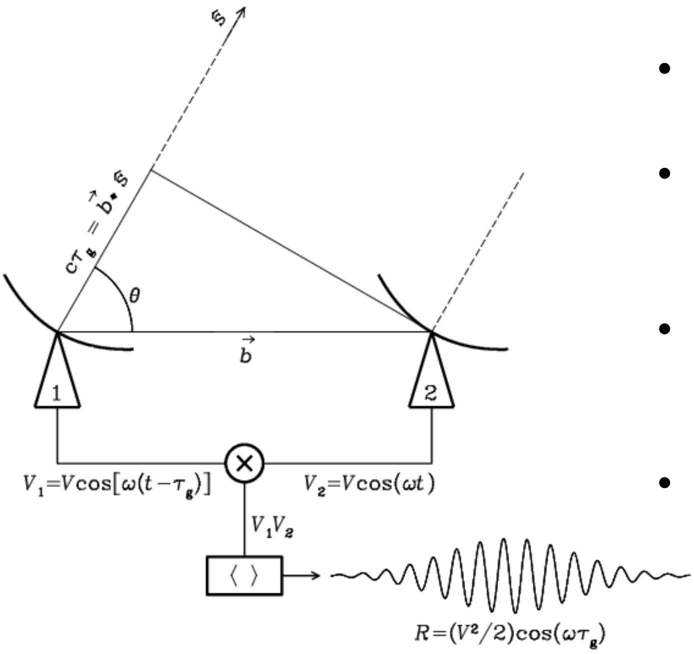
FOV & θ_s of interferometers

- Optical telescopes:
  detectors with millions pixels

- Radio single dish antennas:
  one or a small number of receivers
  (e.g., TRAO SEQUOIA with 16 pixels)

- Interferometers:
  e.g., ALMA 12 m antennas over 12 km in Band 6 (~1.2 mm)
  FOV ~ λ/D ~ 20”
  θ_s ~ λ/(longest baseline) ~ 0.02”
  ==> 10^6 “pixels”

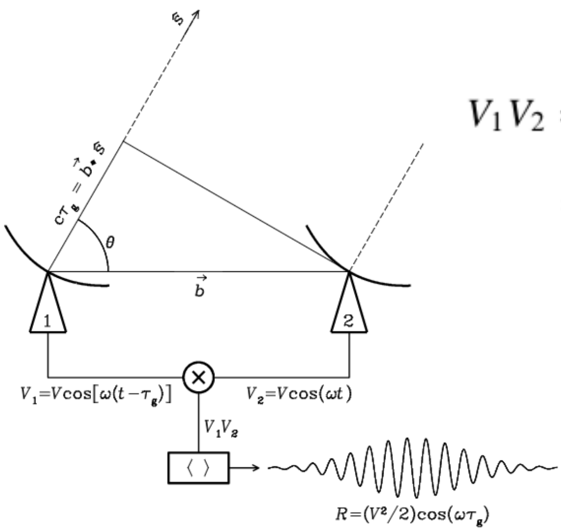
Quasi-monochromatic 2-element interferometer



- "General" case!
- Quasi-monochromatic condition: $\Delta\nu \ll 1/\tau_g$
- **Correlator**: multiply and time-average
- **Geometric delay**: $\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}$

Output of correlator

$$V_1 = V\cos[\omega(t-\tau_g)] \quad \text{and} \quad V_2 = V\cos(\omega t).$$



$$V_1 V_2 = V^2 \cos[\omega(t-\tau_g)]\cos(\omega t)$$
$$= \left(\frac{V^2}{2}\right)[\cos(2\omega t - \omega\tau_g) + \cos(\omega\tau_g)]$$

$\cos x \cos y = [\cos(x+y) + \cos(x-y)]/2$

$\Delta t \gg (2\omega)^{-1}$

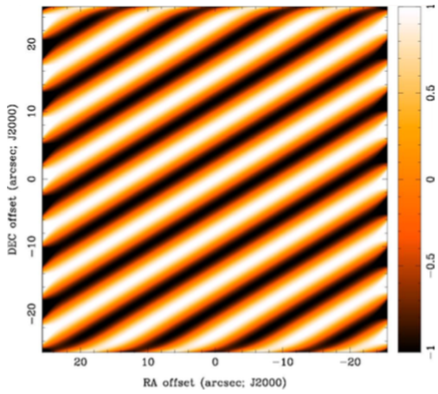
$$R = \langle V_1 V_2 \rangle = \left(\frac{V^2}{2}\right)\cos(\omega\tau_g).$$

Fringes

- Fringes: sinusoidal correlator output
- Fringe phase

$$\phi = \omega \tau_g = \frac{\omega}{c} b \cos\theta$$

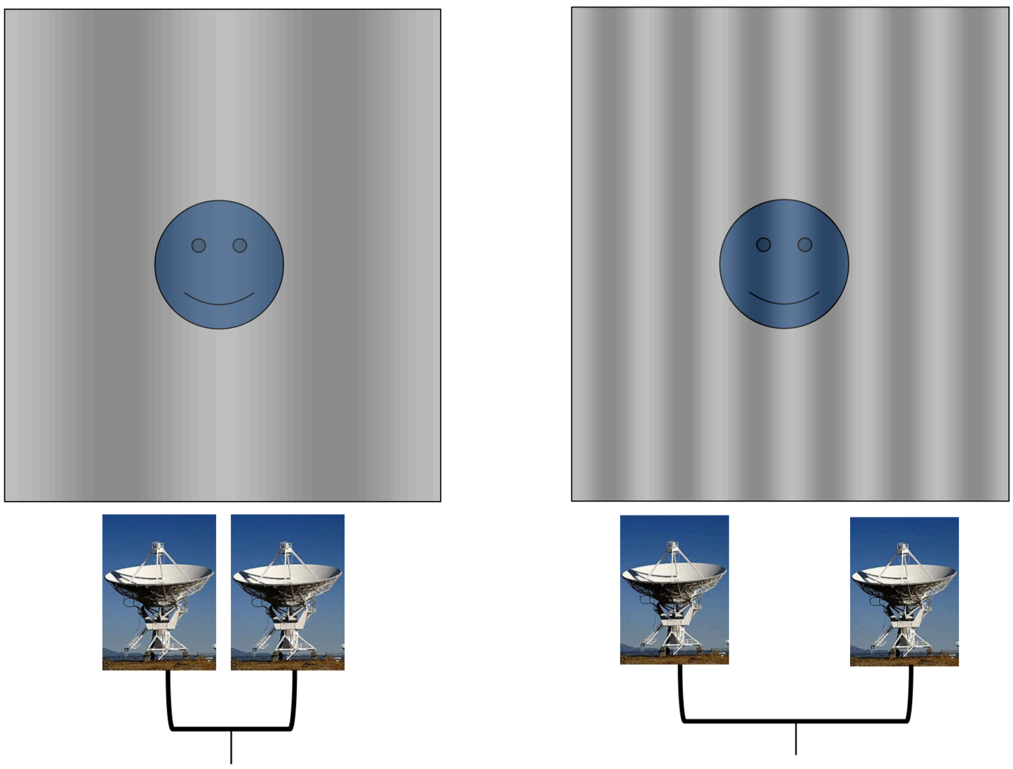
$$\left|\frac{d\phi}{d\theta}\right| = \frac{\omega}{c} b \sin\theta = 2\pi\left(\frac{b\sin\theta}{\lambda}\right)$$



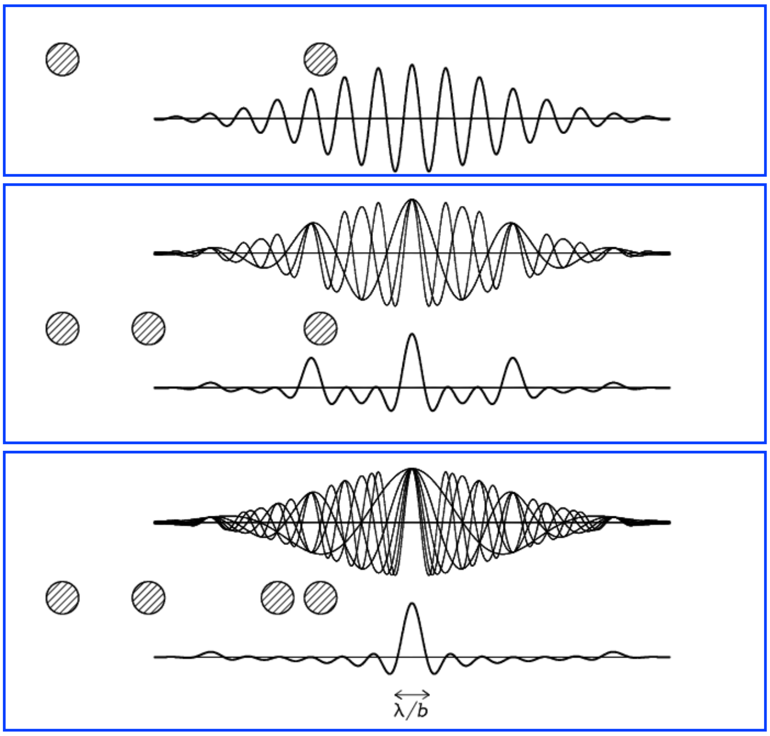
The **fringe period** $\Delta\phi = 2\pi$ corresponds to an angular shift $\Delta\theta = \lambda/(b\sin\theta)$.

Depending on projected baselines
Good image?

Sensitive Scales of Fringes



More antennas better image



Complex correlator

- Slightly extended sources ($I = I_E + I_O$)
  - "cosine" correlator sensitive to even (inversion symmetric) structure
  - "sine" correlator sensitive to odd (anti-symmetric) structure

$$R_c = \int I(\hat{s}) \cos(2\pi\nu \vec{b}\cdot\hat{s}/c) d\Omega = \int I(\hat{s}) \cos(2\pi\vec{b}\cdot\hat{s}/\lambda) d\Omega$$

$$R_s = \int I(\hat{s}) \sin(2\pi\vec{b}\cdot\hat{s}/\lambda) d\Omega$$

- Complex correlator: combination of cosine and sine correlators
  cf. Euler's formula $e^{i\phi} = \cos\phi + i\sin\phi$

- **Complex visibility** $\mathcal{V} \equiv R_c - iR_s$ $\quad \mathcal{V} = Ae^{-i\phi}$ $\quad A = (R_c^2 + R_s^2)^{1/2}$, $\phi = \tan^{-1}(R_s/R_c)$

$$\mathcal{V} = \int I(\hat{s}) \exp(-i2\pi\vec{b}\cdot\hat{s}/\lambda)\ d\Omega$$

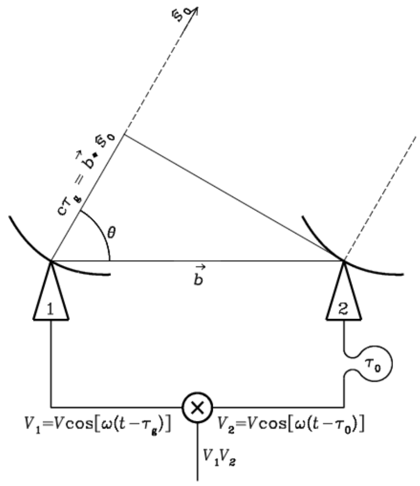
Bandwidth smearing

- Quasi-monochromatic interferometers
  => interferometers with finite bandwidths and integration times

$$\mathscr{V} = \int \left[\int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta\nu/2} I_\nu(\hat{s}) \exp(-i2\pi \vec{b}\cdot\hat{s}/\lambda)\ d\nu \right] d\Omega$$

$$= \int \left[\int_{\nu_c - \Delta\nu/2}^{\nu_c + \Delta\nu/2} I_\nu(\hat{s}) \exp(-i2\pi\nu\tau_g)\ d\nu \right] d\Omega.$$

$$\mathscr{V} \approx \int I_\nu(\hat{s})\ \text{sinc}\ (\Delta\nu\ \tau_g)\ \exp(-i2\pi\nu_c\tau_g) d\Omega.$$



- Instrumental delay τ_0
  to minimize the attenuation

$|\tau_0 - \tau_g| \ll (\Delta\nu)^{-1}$ $\quad$ $\Delta\nu\ \Delta\tau_g \ll 1$ $\quad$ $\Delta\nu\ (b\sin\theta)\ \ \Delta\theta/c \ll 1$

$c\tau_g = \vec{b}\cdot\vec{s} = b\cos\theta$ $\quad$ $|c\Delta\tau_g| = b\sin\theta\Delta\theta$ $\quad$ $\theta_s \approx \lambda/(b\sin\theta)$

$$\Delta\nu \ll \nu\frac{\theta_s}{\Delta\theta}$$

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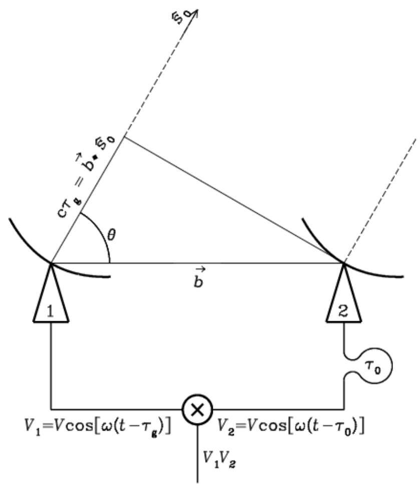


- Instrumental delay τ_0
  to minimize the attenuation

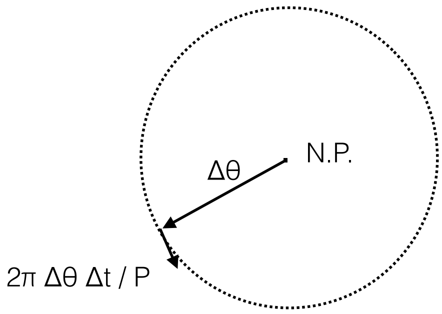
$|\tau_0 - \tau_g| \ll (\Delta$

$c\tau_g = \vec{b}\cdot\vec{s} = b$

$$\Delta\nu \ll \frac{\nu\theta_s}{\Delta\theta} = \frac{1.5\times10^9\ \text{Hz}\cdot 4\ \ \text{arcsec}}{900\ \ \text{arcsec}} \approx 7\ \ \text{MHz}.$$

$$\Delta\nu \ll \nu\frac{\theta_s}{\Delta\theta}$$

Integration time smearing



$$P \approx 23^{\mathrm{h}}56^{\mathrm{m}}04^{\mathrm{s}} \approx 86164$$

$$\frac{2\pi \Delta\theta \Delta t}{P} \ll \theta_s$$

$$\Delta t \ll \frac{\theta_s}{\Delta\theta} \cdot 1.37 \times 10^4 \ \mathrm{s}$$

$$\Delta t \ll \frac{\theta_s}{\Delta\theta} \cdot 1.37 \times 10^4 \ \mathrm{s} = \frac{4 \ \mathrm{arcsec}}{900 \ \mathrm{arcsec}} \cdot 1.37 \times 10^4 \ \mathrm{s} \approx 60 \ \mathrm{s}.$$

Visibility

- Now, assuming an interferometer with a negligible bandwidth attenuation
- Visibility: data of interferometers (Fourier transform of an image)

Steering antenna with an instrumental delay

$$\tau = \tau_g - \tau_i$$

Phase center s_0

$$\hat{s} = \hat{s_0} + \hat{\sigma}$$

$$\mathcal{V} \approx \int I_\nu(\hat{s}) \, \mathrm{sinc}\,(\Delta\nu\,\tau_g) \exp(-i2\pi\nu_c\tau_g) d\Omega.$$

$$V \approx \int A(\hat{s}) I_\nu(\hat{s}) \exp(-i2\pi\nu\tau) d\Omega$$

$$= \int A(\hat{s}) I_\nu(\hat{s}) \exp[-i2\pi\nu\Big(\frac{\vec{b}\cdot\hat{s}}{c} - \tau_i\Big)] d\Omega$$

$$= \exp[-i2\pi\nu(\frac{\vec{b}\cdot\hat{s_0}}{c} - \tau_i)] \int A(\hat{s}) I_\nu(\hat{s}) \exp[-i2\pi\nu\Big(\frac{\vec{b}\cdot\hat{\sigma}}{c}\Big)] d\Omega$$

$$V = \int A(\hat{s}) I_\nu(\hat{s}) \exp[-i2\pi\nu\Big(\frac{\vec{b}\cdot\hat{\sigma}}{c}\Big)] d\Omega$$

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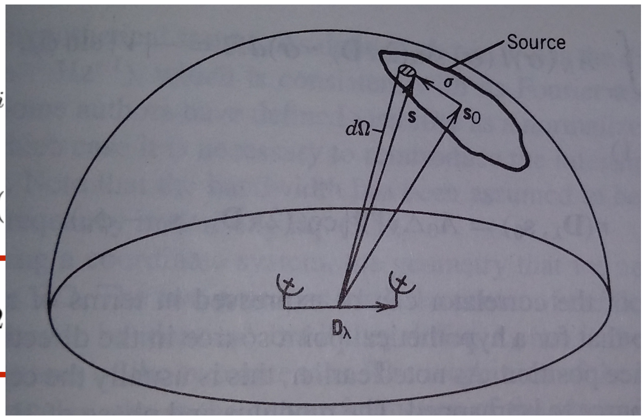
$$= \exp[-i2\pi\nu(\frac{\vec{b}\cdot\hat{s}_0}{c} - \tau_i)] \int A(\hat{s})I_\nu($$

$$V = \int A(\hat{s})I_\nu(\hat{s})\exp[-i2\pi\nu\Big(\frac{\vec{b}\cdot\hat{\sigma}}{c}\Big)]d\Omega$$



Interferometers in 3D

$$V = \int A(\hat{s})I_\nu(\hat{s})\exp[-i2\pi\nu\Big(\frac{\vec{b}\cdot\hat{\sigma}}{c}\Big)]d\Omega$$

$$2\pi\nu\frac{\vec{b}}{c} = 2\pi\frac{\vec{b}}{\lambda} = 2\pi(u,v,w)$$

$$\hat{\sigma} = (l,m,n)$$

$$d\Omega = \frac{dl\,dm}{\sqrt{1-l^2-m^2}}$$

$$V(u,v,w) = \int\int \frac{A(l,m)I(l,m)}{\sqrt{1-l^2-m^2}} \exp[-i2\pi(ul+vm+w\sqrt{1-l^2-m^2})]\,dl\,dm$$

$$\sqrt{1-l^2-m^2} \approx 1$$

$$V(u,v,w) = \exp(-i2\pi w)\int\int A(l,m)I(l,m)\ \exp[-i2\pi(ul+vm)]\,dl\,dm$$

Interferomet

$$V \quad = \quad \int A(\hat{s}) I_\nu(\hat{s}) \exp[-i2\pi\nu\left(\frac{\vec{b}\cdot\hat{\sigma}}{c}\right)$$

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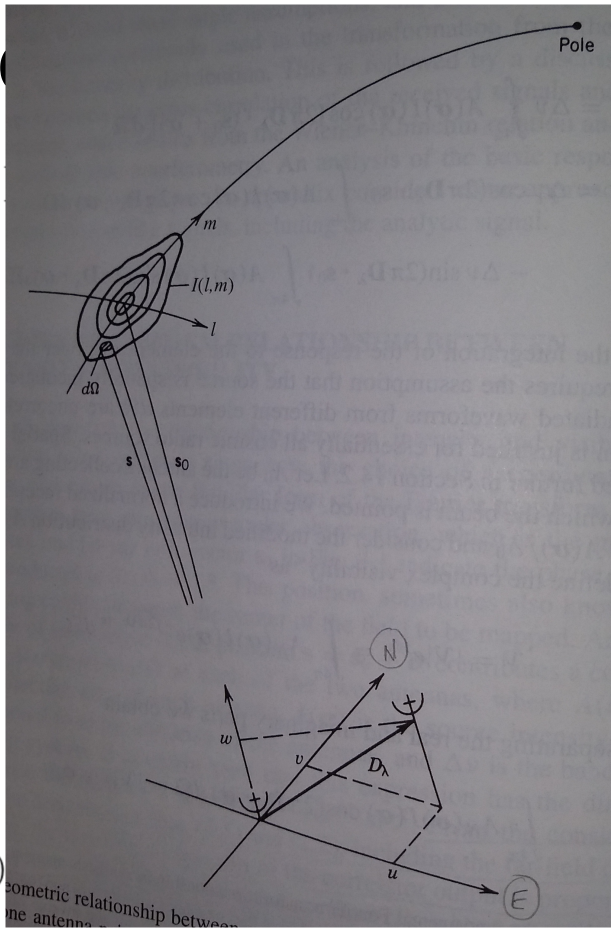


Visibility and Image

- (Inverse) Fourier transformation

On w = 0 plane:

$$V(u,v) = \int\int A(l,m)I(l,m)\ \exp[-i2\pi(ul+vm)]\, dl\, dm$$

$$V(u,v) \quad \rightleftharpoons \quad A(l,m)I(l,m)$$

$$S(u,v)V(u,v) \quad \rightleftharpoons \quad FT^{-1}[S(u,v)] * FT^{-1}[V(u,v)]$$

$$S(u,v)V(u,v) \quad \rightleftharpoons \quad B_D(l,m) * [A(l,m)I(l,m)]$$

Sensitivity

- A single antenna $$\sigma_S = \frac{2kT_s}{A_e(\Delta\nu\,\tau)^{1/2}}$$

- A two-element interferometer $$\sigma_S = \frac{2^{1/2}kT_s}{A_e(\Delta\nu\,\tau)^{1/2}}$$

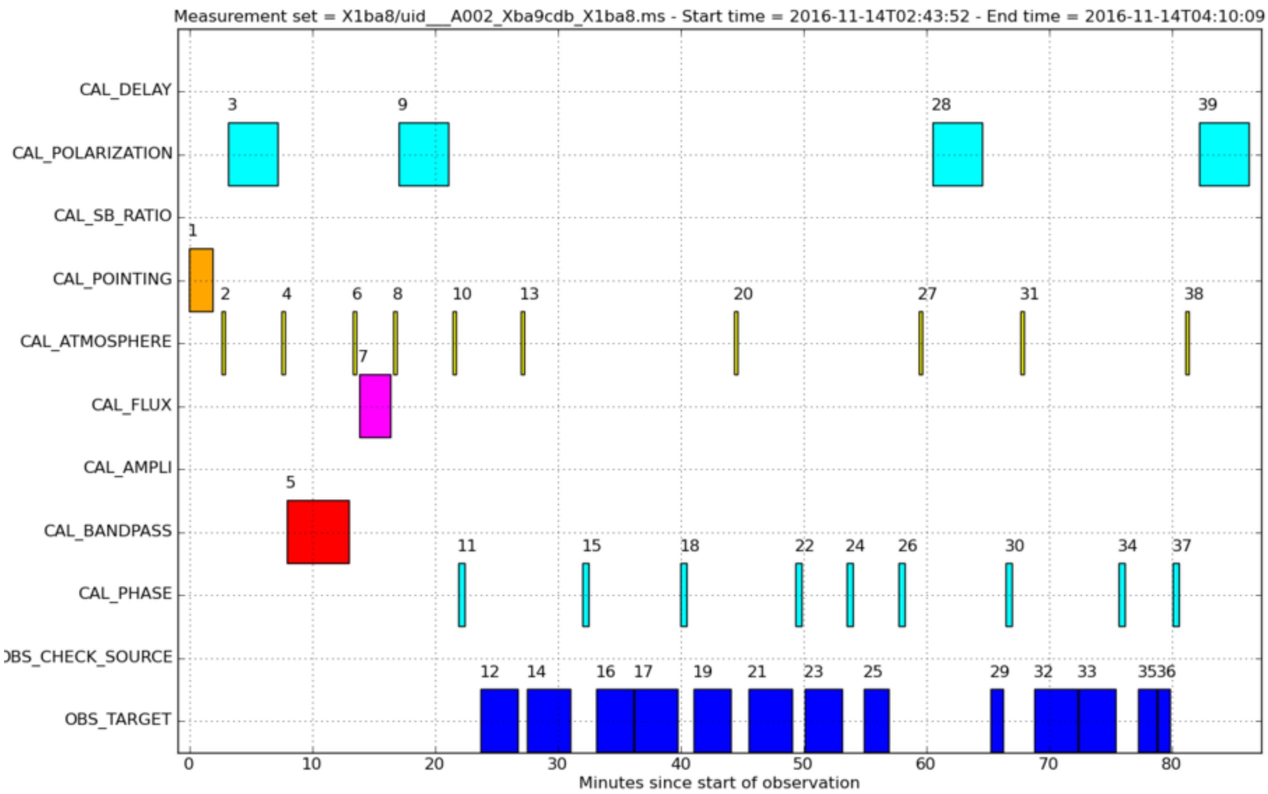
- A N-element interferometer: N(N-1)/2 independent paris

$$\sigma_S = \frac{2kT_s}{A_e[N(N-1)\,\Delta\nu\,\tau]^{1/2}}$$

Interferometric observations

- Calibrators
  Flux (also called amplitude) calibrator
  Bandpass calibrator
  Phase calibrator

- A typical sequence
  Flux cal. —> Bandpass cal. —> Phase cal. and science target cycles (e.g., 10 min period) —> Last phase cal.

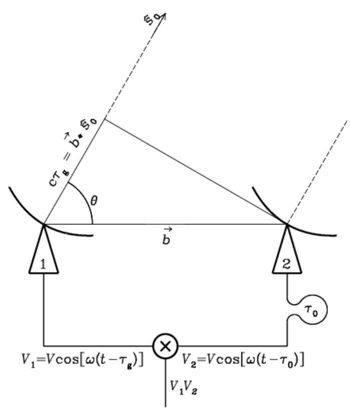


Observing Schedule

From raw data to image

- Calibration: to have all antennas phased up
  - Bandpass calibration
  - Flux (amplitude) calibration
  - Phase calibration

- Imaging: from calibrated visibilities to images
  - Inverse Fourier transform
  - Deconvolution
  - Primary beam correction



$$S(u,v)V(u,v) \rightleftharpoons B_D(l,m) * [A(l,m)I(l,m)]$$

Take-home messages

- Interferometry samples Fourier components of sky brightness: visibilities
- Images are made by Fourier transforming sampled visibilities
  - images are not unique
  - limited scales of detected structures due to missing visibilities

$$S(u,v)V(u,v) \rightleftharpoons B_D(l,m) * [A(l,m)I(l,m)]$$